Properties and Effects of η Decays

RIAZUDDIN
Physics Department, Panjab University, Lahore, Pakistan

AND

FAYYAZUDDIN*
Physics Department, Imperial College, London, England

(Received 25 October 1962)

The partial widths for the $\pi^+\pi^-\pi^0$ and $3\pi^0$ decay modes of the η meson are calculated via an effective electromagnetic vertex $\eta^0 \to \pi^0$, the strength of which is estimated from the electromagnetic violation of the charge independence of nuclear forces. A picture consistent with experiment is obtained. The effect of η on violation of the $\Delta T=\frac{1}{2}$ rule for the nonleptonic decays of K mesons is also considered, and in particular it is shown that the observed ratio for $K^+ \to 2\pi$ versus $K_1^0 \to 2\pi$ can be explained by this picture. Lastly, by comparing total decays of $K_2^0 \to \pi^+\pi^-\pi^0$ and $\Sigma^- \to n+\pi^-$ via the $K-\pi$ weak vertex, the coupling constant $g_{\Sigma NK}^2$ is estimated.

THE assignment of zero spin, odd spatial parity, and even G parity (0^{-+}) to the recently discovered η meson has been suggested by several authors.[1–7] In particular, a recent experiment by Chrétien *et al.*[8] has established the existence of a 2γ decay mode of the η so that the spin of the η is 0 or 2. However, the presently available Dalitz plots[2,9,10] are compatible only with spin 0. The absence of a 2π decay mode then implies odd parity and since $T=0$ for η, the 2γ decay mode implies even G parity.

For the 0^{-+} assignment, decays of η via strong interactions are essentially forbidden[4] and we consider the following electromagnetically permitted decay final states: (a) 3π, (b) 2γ, and (c) $\pi^+\pi^-\gamma$. The experimental indications regarding the partial widths for these modes are[2]

$$\Gamma_\eta(\pi^+\pi^-\gamma)/\Gamma_\eta(\text{all modes})<1/20$$

and[9,10]

$$\Gamma_\eta(\text{all neutral modes})/\Gamma_\eta(\pi^+\pi^-\pi^0)\approx 3,$$

while

$$\Gamma_\eta(3\pi^0)\approx\Gamma_\eta(2\gamma)$$

is not inconsistent with the experiment of Chrétien *et al.*[8] Since all the above decay modes go via electromagnetic interaction, the partial widths are expected to be very small.

The purpose of this note is to calculate the partial widths for the $\pi^+\pi^-\pi^0$ and $3\pi^0$ modes of η via an effective electromagnetic vertex $\eta^0 \to \pi^0$, the strength of which we estimate from the electromagnetic violation of charge independence of nuclear forces. We get a picture consistent with experiment. The effect of η on violation of the $\Delta T=\frac{1}{2}$ rule for the nonleptonic decays of K mesons is also considered, and in particular we show that the observed ratio for $K^+ \to 2\pi$ versus $K_1^0 \to 2\pi$ can be explained on the basis of this picture. Lastly by comparing the decay rates for $K_2^0 \to \pi^+\pi^-\pi^0$ and $\Sigma^- \to n+\pi^-$ via the $K-\pi$ weak vertex, we estimate the coupling constant $g_{\Sigma NK}^2$.

It has been pointed out by several authors[5–7] that the 0^{-+}, $T=0$ assignment for η is unique in predicting a connection between the Dalitz plots for the 3π decay modes of η and K^+, K_2^0. Thus, if both η and K_2^0 decay into 3π's $(T=1)$[11] via a one-pion intermediate state (See Fig. 1.), the Dalitz plots for η decay and K_2^0 decay are determined by the π-3π amplitude. In fact, we may then write[6] the invariant matrix element for K or η decay in

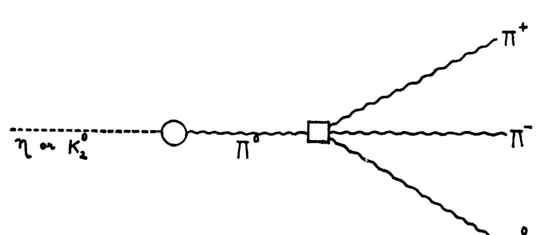

FIG. 1. Feynman diagram for decay of K or η meson through a one-pion intermediate state.

* Present address: Pakistan Atomic Energy Centre, P. O. Box No. 658, Lahore, Pakistan.

[1] A. Pevsner, R. Kvaemev, M. Nussbaum, *et al.*, Phys. Rev. Letters 7, 421 (1961).
[2] P. L. Bastien, J. P. Berge, O. I. Dahl, *et al.*, Phys. Rev. Letters 8, 114 (1962).
[3] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).
[4] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 115, 460 (1962).
[5] G. Barton and S. P. Rosen, Phys. Rev. Letters 8, 414 (1962).
[6] M. A. Baqi Bég, Phys. Rev. Letters 9, 67 (1962).
[7] K. C. Wali, Phys. Rev. Letters 9, 120 (1962).
[8] M. Chrétien, F. Bulos, H. Crouch, *et al.*, Phys. Rev. Letters 9, 127 (1962).
[9] R. Strand *et al.*, in *Proceedings of the 1962 Annual International Conference on High Energy Physics at CERN July, 1962* (CERN, Geneva, 1962).
[10] C. Alff, D. Berley, D. Colley, *et al.*, Phys. Rev. Letters 9, 325 (1962).

[11] The final state of 3π in both η and K decays has $T=1$. This is because η decays into 3π by violating charge independence so that if we consider the nonvanishing lowest order in the electromagnetic interaction, invariance under charge conjugation requires that the final state of 3π's be in the $T=1$ state while in the case of the 3π decay modes of K^+ and K_2^0, the final $T=1$ state is a consequence of the $\Delta T=\frac{1}{2}$ rule and invariance under CP.

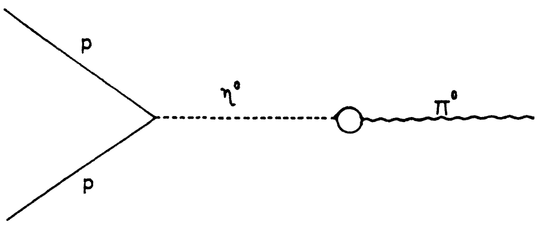

FIG. 2. Feynman diagram for electromagnetic correction to the $pp\pi^0$ coupling constant due to the η meson.

the form

$$M_{i;\,\rho,\alpha,\beta,\gamma}(s_1,s_2,s_3)=\lambda_i[A(s_1,s_2,s_3)\delta_{\rho\alpha}\delta_{\beta\gamma}+B(s_1,s_2,s_3)\delta_{\rho\beta}\delta_{\gamma\alpha}+C(s_1,s_2,s_3)\delta_{\rho\gamma}\delta_{\alpha\beta}],\quad(1)$$

where $i=K$ or η and $s_1=-(K-k_1)^2$, $s_2=-(K-k_2)^2$, $s_3=-(K-k_3)^2$; k_1, k_2, k_3 being the 4-momenta of the three emerging pions while K is the 4-momentum of the K or η meson. At the symmetric point $s_1=s_2=s_3=s_0$,

$$A(s_0,s_0,s_0)=B(s_0,s_0,s_0)=C(s_0,s_0,s_0)\approx-\lambda,$$

where λ is the π-π coupling constant introduced by Chew and Mandelstam.[12] λ_K and λ_η can be expressed as follows:

$$\begin{aligned}\lambda_K&=G_{K\pi}[m_\pi^2/(m_K^2-m_\pi^2)]\\ \lambda_\eta&=G_{\eta\pi}[m_\pi^2/(m_\eta^2-m_\pi^2)],\end{aligned}\quad(2)$$

where $G_{K\pi}$ and $G_{\eta\pi}$ are dimensionless coupling constants characterizing the vertices $K_2{}^0\to\pi^0$ and $\eta^0\to\pi^0$. The various decay spectra are then given as follows[6]:

$$\begin{aligned}|M_\tau|^2&=\lambda_K^2|A+B|^2,\\ |M_{K_2{}^0\to\pi^+\pi^-\pi^0}|^2&=\lambda_K^2|C|^2=|M_{\tau'}|^2,\\ |M_{\eta\to\pi^+\pi^-\pi^0}|^2&=\lambda_\eta^2|C|^2=(\lambda_\eta/\lambda_K)^2|M_{K_2{}^0\to\pi^+\pi^-\pi^0}|^2,\\ |M_{\eta\to3\pi^0}|^2&=\lambda_\eta^2|A+B+C|^2=(\lambda_\eta/\lambda_K)^2|M_\tau+M_{\tau'}|^2.\end{aligned}\quad(3)$$

This provides the required indentification of the Dalitz plots. In fact, if the τ-decay are adequately described by the linear (in the kinetic energy of the unlike pion) fit of Gell-Mann and Rosenfeld,[13] only one parameter is needed to fit the shapes of the various spectra in K^+, $K_2{}^0$, and η decays. Thus, the π^0 spectrum in η decay is identical with that of the π^0 in $K_2{}^0$ decay and of the π^+ in $\tau^{+\prime}$ decay and is described by [5,7,13]

$$M_\eta\sim1-ay,\quad(4)$$

where $y=(T_0-\frac{1}{3}Q)/\frac{1}{3}Q$, $Q=m_\eta-3m_\pi$, $T_0=$kinetic energy of the π^0, and $a\approx1/5$ from τ^+ and $\tau^{+\prime}$ data.[14] The spectrum of the π^0 in η decay thus determined is found to be in reasonable agreement[6,7,10] with the data available. Wali[7] also has shown that the present data indicate that the ratio $R\,[(\eta\to3\pi^0)/(\eta\to\pi^+\pi^-\pi^0)]$ lies between 1.6 and 1.7. If M_η is constant independent of

[12] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).
[13] M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. **7**, 407 (1957).
[14] See reference 7 where other references are given.

energy, then R has its maximum value $\frac{3}{2}(1.15)=1.73$ (1.15 being the phase space factor arising from the π^+-π^0 mass difference).

In calculating the absolute decay rate for η or K going to 3π, we shall consider M_η or M_K to be constant, as the term containing y contributes very little to the total decay rate. For the same reason we shall approximate the quantities A, B, and C introduced in Eq. (1) by their value at the symmetric point, i.e., by λ, the π-π coupling constant. Then the decay widths are given by

$$\Gamma_\eta(\pi^+\pi^-\pi^0)=\frac{\lambda_\eta^2}{\lambda_K^2}\Gamma(K_2{}^0\to\pi^+\pi^-\pi^0)\frac{m_\eta}{m_K}\left(\frac{m_\eta-3m_\pi}{m_K-3m_\pi}\right)^2,\quad(5)$$

or alternatively

$$\Gamma_\eta(\pi^+\pi^-\pi^0)=\frac{\lambda^2}{16\pi^2}\frac{G_{\eta\pi}{}^2}{[(m_\eta/m_\pi)^2-1]^2}\frac{1}{2^3 3\sqrt{3}}\left(1-\frac{3m_\pi}{m_\eta}\right)^2m_\eta,\quad(6)$$

while

$$\Gamma(K_2{}^0\to\pi^+\pi^-\pi^0)=\frac{\lambda^2}{16\pi^2}\frac{G_{K\pi}{}^2}{[(m_K/m_\pi)^2-1]^2}\frac{1}{2^3 3\sqrt{3}}\left(1-\frac{3m_\pi}{m_K}\right)^2m_K.\quad(7)$$

Thus, we see that we can calculate the partial width $\Gamma_\eta(\pi^+\pi^-\pi^0)$ and also $\Gamma_\eta(3\pi^0)$ provided that we know the strength $G_{\eta\pi}$ of the vertex $\eta^0\to\pi^0$, the transition $\eta^0\to\pi^0$ being via electromagnetic interaction. $G_{\eta\pi}$ can in fact be related to the electromagnetic correction to the pion-nucleon coupling constant $g_{\pi NN}$, as is clear from Fig. 2.[15] Fig. 2, we have

$$g_{\eta NN}G_{\eta\pi}m_\pi^2/(m_\eta^2-m_\pi^2)=\delta g_{\pi NN},$$

From where $\delta g_{\pi NN}$ denotes the electromagnetic connection to the π^0pp or π^0nn coupling constant. Therefore

$$G_{\eta\pi}{}^2=\left(\frac{\delta g_{\pi NN}}{g_{\pi NN}}\right)^2\frac{g_{\pi NN}{}^2/4\pi}{g_{\eta NN}{}^2/4\pi}\left(\frac{m_\eta^2}{m_\pi^2}-1\right)^2.\quad(8)$$

There is no corresponding contribution to the $np\pi^+$ vertex so that η violates charge independence in the sense that the $pp\pi^0$ or $nn\pi^0$ coupling constant is different from the $np\pi^+$ coupling constant. Let us now discuss whether there is some experimental evidence for such a difference. In fact, from the experimentally determined[16] values of $g_{\pi^0pp}{}^2$ and $g_{\pi^+np}{}^2$ one cannot exclude a difference of 2 to 3% within the experimental errors. On the other hand, there is some evidence that such a difference ($\delta g_{\pi NN}/g_{\pi NN}\approx1\%$) might very well exist. This evidence

[15] There are no diagrams corresponding to Fig. 2 for ρ and ω (spin 1) mesons because of spin conservation and for ζ (if it exists) because of parity conservation.
[16] G. Breit, M. H. Hull, Jr., K. Lassila, and K. D. Pyatt, Jr., Phys. Rev. Letters **4**, 79 (1960). See also D. Y. Wong and H. P. Noyes, Phys. Rev. **126**, 1866 (1962).

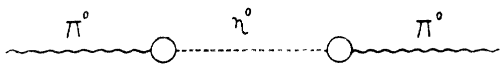

FIG. 3. Feynman diagram for the self-energy of π^0 due to the η meson.

comes from nuclear forces,[17,18] particularly from the difference in the singlet s-wave scattering lengths of np and pp systems.[17,19] Also such a difference might account[20] for the discrepancy of 1–2% between G_V and G_μ,[21] the coupling constants for β and μ decay, respectively, the equality of which is required by the conserved current hypothesis.

From now on we shall take $(\delta g_{\pi NN}/g_{\pi NN})$ to be of the order of 1%. Then η also contributes to the π^0 self-energy according to the Feynman diagram shown in Fig. 3, so that

$$\delta m_{\pi^0}{}^2 = -G_{\eta\pi}{}^2[m_\pi{}^2/(m_\eta{}^2/m_\pi{}^2-1)].$$

With $\delta g_{\pi NN}/g_{\pi NN}\approx 1\%$, $g_{\eta NN}{}^2/4\pi\approx 2$,[22] $g_{\pi NN}{}^2/4\pi\approx 15$, we get $\delta\mu^2/\mu^2=(m_{\pi^+}{}^2-m_{\pi^0}{}^2)/m_\pi{}^2\approx 1\%$, whereas experimentally $\delta\mu^2/\mu^2$ is 7%. Of course the major contribution to the π^+-π^0 mass difference comes from the π^+ self-energy. Marshak and Bose[23] have recently calculated $\delta\mu$ to be 4.1 MeV on the basis of the 2π resonance at 750 MeV in $J=1$, $T=1$, using a formula derived by a dispersion-theoretic approach. The experimental value of $\delta\mu$ is 4.6 MeV so that the small contribution of η to the mass difference is in the right direction.

Having determined $G_{\eta\pi}$ as above, we can now calculate the partial width $\Gamma_\eta(\pi^+\pi^-\pi^0)$ as given by Eq. (6). With $\lambda/4\pi\approx -0.15$,[24] and our estimate (8) for $G_{\eta\pi}{}^2$ (with $\delta g_{\pi NN}/g_{\pi NN}=1\%$, $g_{\pi NN}{}^2/4\pi\approx 15$, and $g_{\eta NN}{}^2/4\pi\approx 2$), we get

$$\Gamma_\eta(\pi^+\pi^-\pi^0)\approx 14 \text{ eV}, \tag{9}$$

so that

$$\Gamma_\eta(3\pi^0)\approx 1.6\times 14 \text{ eV} = 22 \text{ eV}. \tag{10}$$

Let us now consider other modes, namely, (b) and (c). The $\eta\to 2\gamma$ mode is analogous to $\pi^0\to 2\gamma$ and if we scale Γ_{π^0} as (mass)^3 to the η mass, we have [25,4]

$$\Gamma_\eta(2\gamma)=(m_\eta/m_\pi)^3\Gamma_{\pi^0}(2\gamma) \approx 192 \text{ eV}.$$

Hori *et al.*,[22] on the other hand, considered both η^0 and π^0 going to 2γ via a nucleon and antinucleon pair and thus obtain

$$\Gamma_\eta(2\gamma)=(m_\eta/m_\pi)^3\frac{g_{\eta NN}{}^2/4\pi}{g_{\pi NN}{}^2/4\pi}\Gamma_{\pi^0}(2\gamma) \approx 25 \text{ eV}, \tag{11}$$

with $m_\eta=4m_\pi$, $g_{\eta NN}{}^2/4\pi\approx 2$, and $\Gamma_{\pi^0}(2\gamma)\approx 3$ eV. Combining the estimate of Hori *et al.*[22] for $\Gamma_\eta(2\gamma)$ with our estimates (9) and (10) for $\Gamma_\eta(\pi^+\pi^-\pi^0)$ and $\Gamma_\eta(3\pi^0)$, we find

$$\Gamma_\eta(2\gamma)\approx\Gamma_\eta(3\pi^0),$$
$$\Gamma_\eta(\text{neutrals})/\Gamma_\eta(\pi^+\pi^-\pi^0)\approx 3.5,$$

which are consistent with experiment.[8–10] Hori *et al.*[22] as well as Gell-Mann *et al.*[3] and Brown and Singer estimate $\Gamma_\eta(\pi^+\pi^-\gamma)/\Gamma_\eta(2\gamma)$ via $\eta\to\rho+\gamma$, with a virtual ρ^0 which goes to a γ, and found this ratio to be 1/8, consistent with experiment.

If $(\delta g_{\pi NN}/g_{\pi NN})$ is taken to be 0.7%, then the values (9) and (10) for $\Gamma_\eta(\pi^+\pi^-\pi^0)$ and $\Gamma_\eta(3\pi^0)$ are unchanged provided that $g_{\eta NN}{}^2/4\pi\approx 1$. However, then Eq. (11) gives $\Gamma_\eta(2\gamma)\approx 12$ eV so that $\Gamma_\eta(\text{neutrals})/\Gamma_\eta(\pi^+\pi^-\pi^0)\approx 2.4$ which is consistent with experiment,[9,10] but in this case $\Gamma_\eta(2\gamma)$ is different from $\Gamma_\eta(3\pi^0)$. Again with $(\delta g_{\pi NN}/g_{\pi NN})\approx 1\%$ and $g_{\eta NN}{}^2/4\pi\approx 1$, $\Gamma_\eta(\pi^+\pi^-\pi^0)$ and $\Gamma_\eta(3\pi^0)$ become, respectively 28 and 44 eV while $\Gamma_\eta(2\gamma)$ ≈ 12 eV from Eq. (11), so that $\Gamma_\eta(\text{neutrals})/\Gamma_\eta(\pi^+\pi^-\pi^0)$ ≈ 2, consistent with experiment[9,10]; but in this case also $\Gamma_\eta(2\gamma)$ is not equal to $\Gamma_\eta(3\pi^0)$.

Clearly the foregoing figures should be taken as orders of magnitude only but they do present a picture consistent with experiment.

We have seen that η decays only via T-violating modes. This may provide a new interpretation of violation of the $\Delta T=\frac{1}{2}$ rule in nonleptonic decays of K particles as remarked by Pais and Feinberg[26] in connection with ζ particle. Consider, for example, the long-standing problem of the ratio for $K^+\to 2\pi$ versus $K_1{}^0\to 2\pi$, which is 1/500 rather than $\alpha^2\sim 1/20\,000$. Consider the sequence

$$K^+\to\pi^++\eta^0 \searrow \pi^0,$$

the first link here being a weak transition allowed by a pure $\Delta T=\frac{1}{2}$ rule while the second link $\eta^0\to\pi^0$ is electromagnetic and is responsible for the violation of

[17] Riazuddin, Nucl. Phys. **7**, 217 and 223 (1958); **10**, 96 (Erratum) (1959).

[18] R. J. Blin-Stoyle and M. J. Kearsley, Proc. Phys. Soc. (London) **75**, 147 (1960).

[19] M. J. Moravcsik, Ann. Rev. Nucl. Sci. **10**, 291 (1960); H. P. Noyes and M. J. Moravcsik, *ibid.* **11**, 95 (1961).

[20] R. J. Blin-Stoyle and J. Le Tourneaux, Phys. Rev. **123**, 627 (1961).

[21] H. A. Weidenmuller, Phys. Rev. **128**, 841 (1962); R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger, *ibid.* **127**, 583 (1962).

[22] S. Hori, S. Oneda, S. Chiba, and H. Hiraki, Phys. Letters **1**, 81 (1962).

[23] S. K. Bose and R. E. Marshak, Nuovo Cimento **25**, 529 (1962).

[24] B. R. Desai, Phys. Rev. Letters **6**, 497 (1961). The value of λ used in the text has been shown by Desai to give a good fit to the experimental data on $p+d\to He^3+2\pi$. See also M. Jacob, G. Mahouse, and R. Omnès, Nuovo Cimento **23**, 838 (1962). J. Hamilton, P. Menotti, G. C. Oades, and L. J. Vick, Phys. Rev. **128**, 1881 (1962), also find a value of λ from π-N amplitudes consistent with that used in the text.

[25] R. H. Dalitz, Brookhaven National Laboratory BNL-735 (T-264), July, 1962 (unpublished).

[26] G. Feinberg and A. Pais, Phys. Rev. Letters **8**, 341 (1962).

the $\Delta T=\frac{1}{2}$ rule. In this way we get

$$\frac{R(K^+\to\pi^+\pi^0)}{R(K_1^0\to 2\pi)}=f_{K^+\to\eta^0\pi^+}{}^2\left(\frac{\delta g_{\pi NN}}{g_{\pi NN}}\right)^2\left(\frac{g_{\pi NN}{}^2/4\pi}{g_{\eta NN}{}^2/4\pi}\right)\bigg/ f_{K_1^0\to 2\pi}{}^2, \quad (12)$$

where we have used expression (8) for $G_{\eta\pi}$. $f_{K^+\to\eta^0\pi^+}$ and $f_{K_1^0\to 2\pi}$ are the weak-coupling constants for the decays $K^+\to\eta^0+\pi^+$ and $K_1^0\to 2\pi$, respectively, both of which are allowed by a pure $\Delta T=\frac{1}{2}$ rule. But $\eta^0\pi^+$ is a $T=1$ state while the 2π mode in K_1^0 decay is a $T=0$ state, so that if we take

$$f_{K^+\to\eta^0\pi^+}\approx\sqrt{3}f_{K_1^0\to 2\pi},$$

we get

$$\frac{R(K^+\to\pi^+\pi^0)}{R(K_1^0\to 2\pi)}\approx\frac{1}{444},$$

with $g_{\eta NN}{}^2/4\pi$ again equal to 2 and $(\delta g/g)\approx 1\%$. This result is unchanged if $\delta g/g\approx 0.7\%$ and $g_{\eta NN}{}^2/4\pi\approx 1$.

There is now some experimental evidence for the violation of the $\Delta T=\frac{1}{2}$ rule in the 3π decay of K_2^0, and the η can also be responsible for such a violation if we consider the sequence

$$K_2^0\to\eta^0\to 3\pi.$$

However, in the absence of any workable procedure to estimate the strength of the weak vertex $K_2^0\to\eta^0$, we do not give any numerical estimate.

That we have been able to correlate so many different processes through the η meson is a consequence of the quantum numbers 0^{-+}, $T=0$ assigned to the η meson.

Lastly let us consider the total decay rate for $K_2^0\to\pi^+\pi^-\pi^0$ as given by Eq. (7). This can be calculated provided that we know $G_{K\pi}$. One can approximately fix $G_{K\pi}$ if one assumes that the $\Sigma^-\to n+\pi^-$ is dominated by the K pole. Then

$$\Gamma(\Sigma^-\to n+\pi^-)=2\left(\frac{g_{\Sigma NK}{}^2}{4\pi}\right)G_{K\pi}{}^2\frac{P_\Sigma}{[(m_K/m_\pi)^2-1]^2}, \quad (13)$$

where

$$P_\Sigma=\frac{(\Sigma\pm N)^2-\pi^2}{2\Sigma^2}\left[\left(\frac{\Sigma^2-N^2+\pi^2}{2\Sigma}\right)^2-\pi^2\right]^{1/2};$$

the $\pm$ correspond to the cases of scalar and pseudoscalar $K\Sigma N$ coupling, respectively. Eliminating $G_{K\pi}$ between (7) and (13) and using[27] $R(\Sigma^-\to n+\pi^-)=0.6\times10^{10}$ sec^{-1} and $\lambda/4\pi\approx-0.15$, we find for the pseudoscalar coupling constant $g_{\Sigma NK}{}^2/4\pi$ the values 3 to 1.5 according as[28] $R(K_2^0\to\pi^+\pi^-\pi^0)=1.5\times10^6$ sec^{-1} or 3×10^6 sec^{-1}. For scalar $K\Sigma N$ coupling,

$$g_{\Sigma NK}{}^2/4\pi\approx 0.03.$$

We are indebted to Professor P. T. Matthews and Professor A. Salam for encouragement and to Professor M. J. Moravcsik for critical reading of the manuscript. The financial help given by the Pakistan Atomic Energy Commission is gratefully acknowledged. One of us (R) would like to thank W. Walkinshaw and Dr. R. G. Moorhouse for hospitality at Rutherford High Energy Laboratory (N.I.R.N.S.), Harwell where this work was done. The other (F) is grateful to the British Department of Technical Cooperation for the award of a Fellowship under the Colombo Plan.

[27] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962).
[28] G. Alexander, S. P. Almeida, and F. S. Crawford, Jr., Phys. Rev. Letters **9**, 69 (1962).

PHYSICAL REVIEW VOLUME 129, NUMBER 5 1 MARCH 1963

Equivalence of the Brysk Approximation and the Determinantal Method*

DAVID S. FALK
University of Maryland, College Park, Maryland
(Received 18 October 1962)

It is shown that the first-order approximations, for central potential scattering, of Brysk and of the determinantal method are equivalent.

IN a recent paper[1] Brysk has presented a new approximation for scattering from a potential. He obtains this approximation by iterating on the asymptotic expression for the scattered wave in an asymptotically valid equation for the exact wave function. His result, for a spherically symmetric potential, is

$$\tan\delta_l=\frac{-k\int_0^\infty r^2dr\ j_l{}^2(kr)U(r)}{1-k\int_0^\infty r^2dr\ j_l(kr)n_l(kr)U(r)}, \quad (1)$$

* Supported in part by the U. S. Air Force Office of Scientific Research.
[1] H. Brysk, Phys. Rev. **126**, 1589 (1962).